STUDIES ON MIXING. XXIX.*

SPACIAL DISTRIBUTION OF MECHANICAL ENERGY DISSIPATED BY AXIAL MIXER IN A SYSTEM WITH RADIAL BAFFLES**

I. FOŘT, R. NEUGEBAUER and M. PASTYŘÍKOVÁ

Department of Chemical Engineering,
Institute of Chemical Technology, Prague 6

Received March 13th, 1970

This work is dedicated to the question of spacial distribution of mechanical energy dissipation in Newtonian charge mixed by a paddle mixer with inclined blades in the turbulent flow regime. By a general macroscopic balance of mechanical energy is determined the energy dissipated per unit of time in a volume unit of various parts of the charge when the whole system is axially symmetric. Flow rates energies in different parts of the system are calculated from velocity profiles obtained experimentally, and from total pressures measured by a oriented three-holes Pitot tube. It has been found that spacial distribution of mechanical energy dissipated in the charge is considerably heterogeneous. The mixer power output is defined as the energy dissipated in the volume of charge beyond the region formed by the rotating mixer. Practically, ninety per cent of the mixer power output is dissipated in the space below the plane passing through the upper edge of the mixer blades (reduced for the region of rotating mixer) while the remaining ten per cent are dissipated in the space above this plane which space represents two thirds of the charge volume. The energy dissipated per unit of time in the volume unit below the mixer plane is from twenty to fourty-times greater than the energy dissipated in the charge volume above the mixer plane and this ratio increases with the decreasing relative size of the mixer. The hydraulic efficiency of the studied mixer was found to be from sixty to seventy per cent (in dependence on the relative size of the mixer). Results obtained in this work are valid for Re $> 1{\cdot}0 \cdot 10^4$.

In mixing of liquids, the rotating mixer is transfering mechanical energy to the charge which is dissipated into heat. The measurement of this process is given by the specific power-input of the mixer related with regard to the volume of charge (mean specific power-input of the mixer), which is a quantity used also for modelling of mixing devices since it characterizes the mean rate of energy consumption in the charge and, consequently, also the mean intensity of mixing in micro-dimension caused by turbulent motion of liquid. The mean specific power-input of the mixer can be also

* Part XXVIII: This Journal *36*, 1546 (1971).
** Presented at the IIIrd International Congress CHISA, Mariánské Lázně 1969.

considered as the measure of mechanical energy of the system. From values of the mentioned quantity can be thus considered suitability of the given arrangement of the mixed system for the considered operation, eventually the experimental results from the mixing model can be successfully applied to mixing devices. The specific power-input related with regard to the whole charge volume is, however, the mean quantity for the whole volume of the system and is thus only a certain integral factor of mixer ability to transfer energy to the surrounding liquid. In individual parts of the system, the energy dissipated per unit of time in the unit of charge volume, can vary significantly from place to place in dependence on local hydrodynamic conditions. Besides, a considerable part of the mixer power-input dissipates in the space of the rotating mixer so that real values of rate of energy dissipation in the mixed system can in its individual parts differ considerably from the mean specific power-input of the mixer. Therefore was studied the spacial distribution of power-input dissipation of the mixer in the mixed charge. The obtained results then served as a basis for evaluation of expediency of geometric arrangement of the system from the view of spacial homogeneity of the dissipation rate of mechanical energy supplied to the charge by the mixer.

A general macroscopic balance of mass and energy was presented by Bird[1]. Application of the energy balance to a given limited part of the system with mechanical mixer was given by Nagata and coworkers[2,3]. The cited authors were balancing the mechanical energy in cylindrical space formed by the rotating mixer on basis of experimental study of velocity field in vicinity of turbine or blade mixer with vertical blades. They carried out experiments with Newtonian charge mixed both in the system with and without radial baffles in the turbulent flow regime. They expressed the results of their experiments as portions of energy dissipated per unit of time in space of the rotating mixer and in the remaining part of the charge. It was found that in the system without baffles, more than 2/3 of the mixer power-input was dissipated in the space formed by the rotating mixer, in the system with radial baffles there were dissipated only 14 per cent of the mixer power-input. The authors did not pay attention to spacial distribution of rate of energy dissipated in the mixed system nor did they check whether their experimental data were in agreement with the continuity equation for the given region. From the results of their experiments follows that the mixed system, divided only into two parts (space of rotating mixer and the remaining charge) is from the view of energy dissipation per unit of time in unit of the system volume very heterogeneous.

THEORETICAL

Let us consider a system with six-bladed paddle mixer with blades inclined at 45°, axially situated in a cylindrical vessel with four radial baffles (see Fig. 1). The mixer rotates so as the blades are forcing the liquid toward the vessel bottom. We are introducing the following simplifying assumptions:

1. the system is axially symetric, 2. flow has a turbulent character in the whole system, and 3. the flow regime is stationary.

We define in the mixed system three regions (see Fig. 2, in which these regions are distinguished by different hatching intensities): *1*. region having the form of a truncated cone of volume V_m, limited by horizontal cross-sections *I* and *II* and by conical area between the two considered cross-sections which are the loci of points of reversal of the flow direction from upward to downward. *2*. Cylindrical region of volume V_I limited by the vessel bottom and wall and by cross-section *II*, reduced for volume. of the cone V_m, and 3. cylindrical region of volume V_{II} limited by cross-section *II*, the vessel wall and by the practically still charge surface.

For regions V_I and V_{II} at the assumptions made above the macroscopic balance of mechanical energy can be written in the form

$$\dot{E}_{I1} = \dot{E}_{I2} + \varepsilon_I V_I \,, \tag{1}$$

for region V_I, where

$$\dot{E}_{I1} = 2\pi \int_0^{r_{I1}} [(E(r)]_{I1} \, [\bar{w}_{ax}(r)]_{I1} \, r \, dr \tag{2}$$

and

$$\dot{E}_{I2} = 2\pi \int_{r_{II1}}^{D/2} [E(r)]_{II2} \, [\bar{w}_{ax}(r)]_{II2} \, r \, dr \tag{3}$$

and for the region V_{II} in the form

$$\dot{E}_{II1} = \dot{E}_{II2} + \varepsilon_{II} V_{II} \,, \tag{4}$$

where

$$\dot{E}_{II1} = 2\pi \int_{r_{II2}}^{D/2} [E(r)]_{II2} \, [\bar{w}_{ax}(r)]_{II2} \, r \, dr \tag{5}$$

$\dot{E}_{II1} = \dot{E}_{I2}$

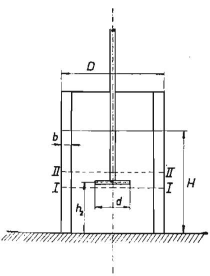

FIG. 1
Schematic View of the Mixed System

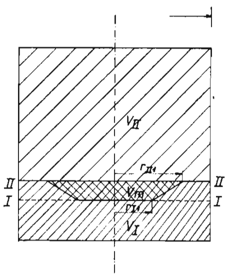

FIG. 2
Determination of the Volumes in the Mixed System

and

$$\dot{E}_{II2} = 2\pi \int_0^{r_{II1}} [E(r)]_{II1} \, [\overline{w}_{ax}(r)]_{II1} \, r \, dr \,, \tag{6}$$

when the only contribution of energy transfer into the considered regions is the flow rate through cross-sections *I* and *II* in the vessel.* In equations (*1*) and (*4*), the symbol ε_i ($i = I, II$) is the energy dissipated per unit of time in the unit of i-th charge volume. Quantity E_{ij} expresses the energy of volume unit in the liquid flowing through the i-th cross-section vertically downwards ($j = 1$), or vertically upwards ($j = 2$). Its radial profile can be explicitly expressed in a form

$$[E(r)]_{ij} = [\bar{p}_{st}(r)]_{ij} + \varrho/2\{[\overline{w}^2(r)]_{ij}\} + z_i \varrho \boldsymbol{g} \,, \quad [i = \mathrm{I}, \mathrm{II} \,; \;\; j = 1, 2] \,. \tag{7}$$

The continuity equation can be written for region V_I in the form

$$2\pi \int_0^{r_{I1}} [\overline{w}_{ax}(r)]_{I1} \, r \, dr = 2\pi \int_{r_{II1}}^{D/2} [\overline{w}_{ax}(r)]_{II2} \, r \, dr = \dot{V}_{cI} \tag{8}$$

and for region V_{II} in the form

$$2\pi \int_{r_{II1}}^{D/2} [\overline{w}_{ax}(r)]_{II2} \, r \, dr = 2\pi \int_0^{r_{II1}} [\overline{w}_{ax}(r)]_{II1} \, r \, dr = \dot{V}_{cII} \,, \tag{9}$$

because neither through the vessel wall nor throught the liquid surface mass enters or leaves and the charge can be considered incompressible. From equation (*8*) and (*9*) follows that total volumetric flow rates through boundaries of both studied systems are the same, *i.e.* that holds

$$\dot{V}_{cI} = \dot{V}_{cII} = \dot{V}_c \,. \tag{10}$$

The quantity

$$[\overline{w}_{ax}(r)]_{ij} \,, \quad [i = I, II_{ij} = 1, 2] \,,$$

expresses the radial profile of the axial component of local velocity, *i.e.* the component vertical to the cross-section *I* or *II*. The term "local velocity" will hereinafter always denote a stationary quantity, *i.e.* quantity averaged to sufficiently long time interval *T*, defined by relation

* From definitions of system boundaries V_I and V_{II} (see Fig. 2) is obvious that quantities $\dot{E}_{I2}$ and $\dot{E}_{II1}$ must be identical.

$$\overline{w_{ij}}(r, z_i) \equiv (1/T) \int_{-T/2}^{T/2} w_{ij}(r, z_i, \tau)\, d\tau\,. \tag{11}$$

Index j then denotes whether the sense of direction of liquid through the i-th cross-section is downwards ($j = 1$) or upwards ($j = 2$).* Value of the integral limit r_{iI} ($i = I, II$) determines the position of this point on the velocity profile in the i-th cross-section where the rate component $\overline{w}_{ax}$ changes its sign; the flow direction in the considered cross-section is reversed.

For region V_m, at the assumptions made, the macroscopic balance of mechanical energy can be written in the form

$$\dot{E}_{II2} + e_{sp}\dot{V}_c = \dot{E}_{I1}\,, \tag{12}$$

where e_{sp} is energy supplied by the mixer to the volume unit of liquid flowing with the volumetric flow rate $\dot{V}_c$ between cross-sections I and II, reduced for energy dissipated in the space of volume $\dot{V}_m$. Volumetric flow rate through the considered region is defined by the continuity equation

$$2\pi \int_0^{r_{II1}} [\overline{w}_{ax}(r)]_{II1}\, r\, dr = 2\pi \int_0^{r_{I1}} [\overline{w}_{ax}(r)]_{I1}\, r\, dr = \dot{V}_c\,. \tag{13}$$

Indices of quantities in equations (*13*) and (*12*) have the same meaning as in mass and energy balances in systems V_I and V_{II}.

The energy is thus gained by liquid flowing through the volume of truncated cone V_m, with bases of diameters r_{I1} a and r_{II1} (Fig. 2). Volume V_m will be further on called the rotor region. By using quantity e_{sp} can be determined the mixer power-output, *i.e.* the energy dissipated per unit of time in the mixed system beyond the volume V_m according to

$$N_t = e_{sp}\dot{V}_c\,, \tag{14}$$

the total height of the mixer by

$$h = e_{sp}/\varrho g \tag{15}$$

and the hydraulic mixer efficiency by

$$\eta_h = N_t/N\,, \tag{16}$$

where N is the power-input of the mixer.

---

* We introduce the negative sign for the sense of direction of flow downwards and the positive sign for flow upwards.

So far introduced hydraulic characteristics of the mixed system can be expressed in dimensionless form. We define the dimensionless mixer power-output by

$$\mathrm{Eu}_t \equiv N_t/\varrho n^3 d^5 \,, \tag{17}$$

the dimensionless total height by

$$H \equiv hg/d^2 n^2 \,, \tag{18}$$

and with regard to definition of the Euler number for mixing

$$\mathrm{Eu}_M \equiv N/\varrho n^3 d^5 \tag{19}$$

the hydraulic mixer efficiency is

$$\eta_h \equiv \mathrm{Eu}_t/\mathrm{Eu}_M \,. \tag{20}$$

Similarly, the dimensionless energy dissipated per unit of time in the volume V_i can be defined by relation

$$\mathrm{Eu}_{z_i} \equiv \varepsilon_i V_i/n^3 d^5 \,, \quad [i = I, II] \tag{21}$$

and the total flow rate $\dot{V}_c$, by use of the flow criterion

$$K_{c_i} \equiv \dot{V}_{ci}/nd^3 \,, \quad [i = I, II] \,. \tag{22}$$

For determining of quantities E_i and e_{sp} it is necessary to get experimentally the radial profiles of quantities $[E(r)]_{ij}$ and $[\overline{w}_{ax}(r)]_{ij}$ as well as the integration limits r_{i1}, *i.e.* points where flow in the considered cross-section reverses its direction. A suitable device for experimental determination of the mentioned quantities is a multi-holes oriented Pitot tube.

EXPERIMENTAL

Measurements were made in a cylindrical vessel with flat bottom of diameter $D = 290$ mm (see Fig. 1) provided with four radial baffles of width $b = 0{\cdot}1$ D. The mixed liquid was distilled water at 20°C ± 1°C. In the experiments were used six-bladed paddle mixers with blades inclined under 45° [4,5] with relative size d/D equal to one fifth, one fourth, and one third. The mixer was situated in the vessel axis and always rotated so that the liquid was forced by the blades toward the vessel bottom. Height of surface of the mixed charge when at rest was equal to the vessel diameter D and the height h_2 of the mixer above the vessel bottom was one fourth of the vessel diameter.

The mixer drive consisted of a direct-current electric motor with a power-input 0·4 kW. The rotational speed could be set by the magnetic controller within the range of 300—2 500 r.p.m. The controller was fed by alternating current from the mains. The rotation speed of the mixer was measured by photoelectric speedometer[6] with an accuracy of 1 revolution and it did not fluctuate by more than $\pm 1\%$ of the set value in the whole measurement.

The measurement of radial profile $[E(r)]_{ij}$ of the energy of unit volume in stream of the mixed liquid, as well as of the radial profile $[\overline{w}_{ax}(r)]_{ij}$ of local velocity, component vertical to the horizontal cross-section *I* or *II*, were made by three-holes Pitot tube by procedure described in the preceding paper[6] of this series. The measurements were made in two radial rays (see Fig. 1). They were: ray *I* — vertical distance 3 mm from the lower edge of the mixer blades, ray *II* — vertical distance 10 mm from the upper edge of mixer blades. The measurements were made always only in one ray in the cross-sections *I* and *II*: in the radial ray passing through the axis of the mixed system and the axis of the circular segment between two neighbouring baffles. To acquire more general results, the measurements were made at three rotational speeds of the mixer.

Flow of liquid in vicinity of a rotating mixer does not take place only in the vertical plane passing through the axis of the mixed systems, because of significant effect of tangential velocity component caused by the mixer rotation. Therefore, the equipment for shifting the Pitot tube in the mixed system was constructed so as to enable turning of the tube from the mentioned vertical into the tangential direction. The turning was made by use of an eccentre in which was clamped the guide rod of the Pitot tube. During the turning of the eccentre the hole of the middle tube of the Pitot tube was in the same point and there changed only the angle betweeen the hole axis and the vertical axis of the mixed system. Other parts of the measuring device as well as the measuring procedure in the flowing mixed liquid both in the downward and upward directions remained the same as in the already mentioned work[6] of this series.

RESULTS AND DISCUSSION

Profile of the Local Velocity Vector in the Mixer Plane

By evaluation of radial profile of total pressures (described in the above mentioned work[6]) measured by three holes of the used Pitot tube in the cross-sections *I* and *II* were obtained the radial profile of direction and magnitude of the local velocity vector in the studied diameters. Direction of the local velocity vector was given by two angles: angle φ between this vector and horizontal plane, and by the angle ψ characterizing the displacement of the local velocity vector into the tangential direction from the vertical plane containing the system axis. The value of angle ψ was determined by turning the probe into a position in which the manometer reading reached the maximum in the measured point at given conditions. Value of angle φ was calculated from pressure differences measured in the given point by individual tubes of the Pitot tube based on dependence obtained by the tube callibration[6]. Axial component of the local velocity vector was then, for the found values of angles ψ and φ and for the calculated absolute value of the local velocity vector* $\overline{w}$ calculated in a given

* In reality, instead of the quantity $\overline{w}$ was determined the quantity $\overline{w}(1 + \overline{w'^2}/\overline{w}^2)^{1/2}$. Value of coefficient at the absolute local value, in the mean time velocity differs from one the more the higher is the intensity of turbulence $(\overline{w'^2})^{1/2}/\overline{w}$. So far as this quantity is less than 35%, value of the mentioned coefficient is less than 1·05 which is within the limits of accuracy for the used method of determination of size and direction of the local velocity vector from pressure data of the three-holes Pitot tube[16].

point by relation

$$\bar{w}_{ax} = \bar{w} \sin \varphi \cos \psi \,, \tag{23}$$

respectively, in a dimensionless form, related to the circumferential tip velocity of the mixer blades

$$W_{ax} = \bar{w}_{ax}/\pi dn \,. \tag{24}$$

The found values of angles ψ and φ, or their radial profiles correspond to ideas on the liquid flow in vicinity of the axial mixer in a cylindrical system with radial baffles. In the flow directed upwards was in both cross-sections *I* and *II* found the value of angle ψ to be zero, *i.e.* the flow is there already directed by the baffles and it does not comprise the tangential component. In the flow downwards above the

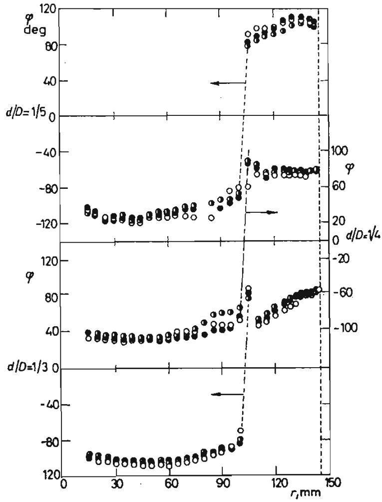

FIG. 3

Radial Profile of Direction of Local Velocity Vector in Cross-section *II* Above the Mixer

d/D	1/3	1/3	1/3	1/4	1/4	1/4	1/5	1/5	1/5
n min^{-1}	350	500	575	600	900	1 100	900	1 100	1 300
point	○	◑	●	○	◑	●	○	◑	●

space V_m, is already evident the effect of mixer rotation with value of angle ψ equal 10° which is constant over the whole cross-section of the flow entering the volume V_m and independent on relative size of the mixer. Effect of the mixer rotation becomes more evident in the flow leaving the space V_m which is proved by the found value of angle $\psi = 30°$. This value is practically constant over the width of the whole flow leaving the space V_m and is independent of both the relative size of the mixer and its speed of rotation. In Fig. 3 and 4, are plotted the dependences of angle φ on the radial distance r in two studied cross-sections (so-called radial profiles of angle φ) for the used mixers and their rotational speeds. In these plots, the regions of negative values of angle φ denote those parts of the system where the liquid flows downward toward the vessel bottom and regions of positive value of angle φ denote those parts of the system where the liquid flows upward toward the liquid surface. Points of zero value of angle φ (positions of reversal of flow direction in the given

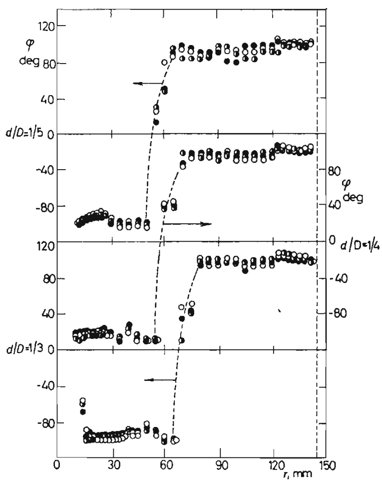

FIG. 4

Radial Profile of Direction of Local Velocity Vector in Cross-section *I* Below the Mixer

d/D	1/3	1/3	1/3	1/4	1/4	1/4	1/5	1/5	1/5
n min^{-1}	350	500	575	600	900	1 100	900	1 100	1 1300
point	○	◑	●	○	◑	●	○	◑	●

cross-section) was found with an accuracy ± 1 mm. From these plots follows that the projection of profile of the local velocity vector direction into the vertical plane corresponds in both studied cross-sections to considerations made on the field of streamlines in vicinity of a rotating axial mixer resp. the vessel wall between two neighbouring baffles. The prevailing flow direction is purely axial. The radial component is significant in the space above the mixer plane (see Fig. 3) in vicinity of the boundary of the downward and upward flow directions. Here due to the suction effect of the mixer originates a flow into the rotor region even from places of much greater radial distance *r* from the system axis, than is the mixer radius and further the whole cross-sectional area of the flow betweeen the cross-sections *I* and *II* is uniformly filled by the liquid flowing upwards. The corresponding part of the cross-section *II* is uniformly filled by the mentioned flow, while in the cross-section *I* in the region of the upward flow exists a relatively large region having a form of a hollow cylinder in which the flow velocity is negligible as compared to the velocity in a narrow region at the vessel wall and at the adjacent radial baffles (see Fig. 5 and 6). In the space below the rotating mixer, the radial component of the local velocity vector appears only in vicinity of the inside boundary of the upward flow, where already takes place a successive reversal of flow directions of liquid layers from downward to up-

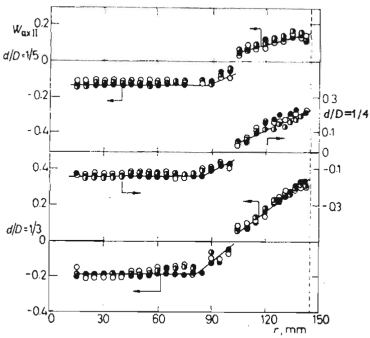

FIG. 5

Radial Profile of Dimensionless Axial Component of Local Velocity Vector in Cross-section *II* Above the Mixer

d/D	1/3	1/3	1/3	1/4	1/4	1/4	1/5	1/5	1/5
n min^{-1}	350	500	575	600	900	1 100	900	1 100	1 300
point	○	◑	●	○	◑	●	○	◑	●

ward. Position of the boundary between the upward and downward flow in the cross-section above the rotor region does not practically depend on the relative size of the mixer and vessel, however, in the cross-section below the rotor region it increases with increasing ratio d/D. From the given figures can be also seen that the found radial profile of angle φ does not depend on the rotational speed of the mixer, which corresponds to until now known facts[6,7] of independence of the flow pattern of the mixed charge on Reynolds number for turbulent flow in the system.

In Fig. 5 and 6 are plotted velocity profiles of the axial component of local velocity vector in dimensionless form in the studied two cross-sections for the used mixers and their rotational speeds. These figures confirm the already experimentally proved[6,7] fact that the axial component of local velocity vector of the mixed liquid depends on first power of rotational speed of the mixer. From the shapes of velocity

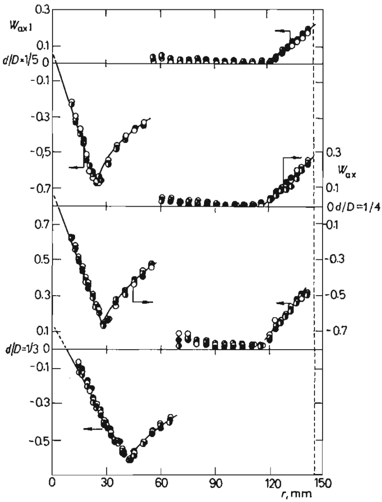

FIG. 6

Radial Profile of Dimensionless Axial Component of Local Velocity Vector in Cross-section *I* Below the Mixer

d/D	1/3	1/3	1/3	1/4	1/4	1/4	1/5	1/5	1/5
n min^{-1}	350	500	575	600	900	1 100	900	1 100	1 300
point	○	◑	●	○	◑	●	○	◑	●

profiles is obvious that the highest flow velocity in the mixed system is in the stream leaving the rotor region and in the region close to the vessel wall where the charge flows upward to the liquid surface. In the space below the mixer plane exist between the mentioned streams "dead regions" of a considerable volume in which the flow velocity is by a factor of orders less than velocity of the stream leaving the rotor region or velocity at the vessel wall. In the space above the rotor region, the stream entering the mentioned region together with the upward flow covers uniformly the whole cross-section *II*. The losses caused by friction in the liquid stream are proportional to the second power of the mean velocity over the considered cross-sectional area of the flow. It is, therefore, energetically most advantageous, if the whole cross-sectional area of the vessel is uniformly filled by the both considered flows so that the mean rates over the both cross-sectional areas are equal. Than, due to equality of total volumetric flow rates in directions downward and upward in the cross-section of vessel *II*, the cross-sectional areas of the upward and downward

TABLE I

Velocity Profile in the Stream at the Inlet and Outlet from Rotor Region

d/D (–)	W_{axI} (Eq. (*26*))	W_{axII} (Eq. (*27*))	$r_{Ic} \cdot 10^3$ m	$r_{I1} \cdot 10^3$ m	$-2/d$ m^{-1}
		Cross-section *I*			
1/5	$-31{\cdot}18r + 0{\cdot}0652$ (0·990)	$-0{\cdot}01716r^{-0.983}$ (0·996)	24·0	53·0	34·50
1/4	$-26{\cdot}02r + 0{\cdot}0820$ (0·996)	$-0{\cdot}01778r^{-1.012}$ (0·993)	28·0	58·0	27·50
1/3	$-18{\cdot}67r + 0{\cdot}1451$ (0·992)	$-0{\cdot}02618r^{-0.984}$ (0·987)	41·5	68·0	20·70

d/D (–)	W_{axII} (Eq. (*28*))	W_{axII} (Eq. (*29*))	$r_{IIc} \cdot 10^3$ m	$r_{II1} \cdot 10^3$ m
		Cross-section *II*		
1/5	$-0{\cdot}1263$ (0·0129)	$4{\cdot}5867r - 0{\cdot}5232$ (0·814)	86·5	103·0
1/4	$-0{\cdot}1383$ (0·0124)	$5{\cdot}979r - 0{\cdot}6461$ (0·874)	85·0	103·0
1/3	$-0{\cdot}1836$ (0·0209)	$7{\cdot}609r - 0{\cdot}8080$ (0·813)	81·5	103·0

flows should be the same, and should be equal to one half of cross-section of the mixed vessel. For radius of the boundary circle between the considered cross-sections of flow should be valid the relation

$$r_{\mathrm{II1}} = \sqrt{2}D/4 = 0{\cdot}1025\ \mathrm{m}\,, \tag{25}$$

which is in a very good agreement with the experimentally found value of the radius r_{II1} of the boundary circle between the upward and downward flow regions in the cross-section *II* (see Table I).

Profile of the quantity W_{ax} at the inlet and outlet of the rotor region was calculated for the given relative mixer size from the measured data by the least square method. In the stream at the outlet from the rotor region were assumed the following forms of velocity profiles:

$$W_{\mathrm{axI}} = k_{\mathrm{I}}r + q_{\mathrm{I}}\,, \quad [r \in \langle 0; r_{\mathrm{Ic}}\rangle]\,, \tag{26}$$

$$W_{\mathrm{axII}} = K_{\mathrm{I}}r^{\mathrm{b}}\,, \quad [r \in \langle r_{\mathrm{Ic}}; r_{\mathrm{I1}}\rangle]\,. \tag{27}$$

In the stream at the inlet into the rotor region were assumed the following forms of velocity profiles

$$W_{\mathrm{axII}} = K_{\mathrm{II}}\,, \quad [r \in \langle 0; r_{\mathrm{IIc}}\rangle]\,, \tag{28}$$

$$W_{\mathrm{axII}} = k_{\mathrm{II}}r + q_{\mathrm{II}}\,, \quad [r \in \langle r_{\mathrm{IIc}}, r_{\mathrm{II1}}\rangle]\,. \tag{29}$$

Table I gives values of constants of regression relations (*26*)–(*29*), calculated from measured data, as well as the radii of circles r_{ic} and r_{ij} ($i = I, II$; $j = 1, 2$), *i.e.* the limits of validity of individual equations. In the mentioned table it is also given (always in the brackets below the calculated regression equation) the correlation coefficient of the given equation eventually the standard deviation of the calculated constant value W_{axII}. In Fig. 5 and 6 the calculated relations are plotted in solid or dashed lines. From the given Figures is obvious the justified use and correctness of the presented regression relations. From the obtained velocity profiles is most interesting the profile of the axial velocity component at the exit from the rotor region. The form of the profile corresponds to the already[8] found course of dependence of the axial velocity component on distance from the axial mixer axis in the stream leaving its blades: it is an analogy of the Rankin vortex caused by action of the inclined mixer blades on the charge in the internal region of a co-axial cylinder with the mixer axis in which the profile of axial velocity component is linear – and the external region of a hollow cylinder bound with the first region, in which the axial velocity component is indirectly proportional to the radial distance from its axis. This fact can be seen very well from the quantitative results given in Table I. Certain deviation originates only in the velocity profile in the internal cylinder where is evident a considerable shift of the profile in respect to the point [0; 0], *i.e.* the straight-line is not passing through the origin. At negligble hub diameter as compared to the mixer diameter the velocity vector should have a zero value in the mixer axis where the rotational velocity is zero. The mentioned deviation is thus caused by the action of the mixer hub, the cross-section of which cannot be neglected for the used mixers, as compared to the cross-section of the mixer, vertical to the rotation axis and formed by the circumcircle to rotating blades of the mixer. From the work[8] of this series follows that the slope of linear part of profile of the dimensionless axial velocity component in the stream leaving the rotor region is

$$k_{\mathrm{I}} = -(2/d) \operatorname{cotg} \gamma \,, \tag{30}$$

which for blades inclined under the angle $\gamma = 45°$ transformes into relation

$$k_{\mathrm{I}} = -2/d \,. \tag{30a}$$

From Table I follows that the mean deviation between the measured and theoretical value of the slope k_{I} is 8·3%.

Total Volumetric Flow Rate in the Mixer Plane

From the determined radial profiles of axial components of local velocity vector were calculated the total flow rates of the mixed liquid in cross-sections *I* and *II* in upward and downward directions by numerical integration of Eq. (*8*) and (*9*). The calculated quantities were also expressed in a dimensionless form by use of the flow criterion K_c (see Eq. (*22*)). Results of the mentioned calculations are given in Table II, where the downward flow has a negative sign, and the upward flow has a positive sign. From results in this Table is evident that within the accuracy of the experiments validity of Eq. (*13*) was proved, *i.e.* that the flow rate of liquid entering the rotor region per unit of time is the same as the volumetric flow rate of the stream leaving the rotor region.* The said conclusion is also evident from comparison of parameters of power relations

$$K_{\mathrm{cI}} = C_1(d/D)^{a_1} \tag{31}$$

and

$$K_{\mathrm{cII}} = C_2(d/D)^{a_2} \,, \tag{32}$$

* This fact, among others, confirms correctness of the used experimental technique since by its use consistent results with general physical laws were obtained.

which were evaluated from the results given in Table II by the least square method and which are given in Table III together with estimates of their standart deviations. Relations (*31*) and (*32*) express dependence of the flow criterion K_c on ratio d/D for flow rates in cross-sections *I* and *II* in both upward and downward directions.

TABLE II

Flow Rates of Mass and Energy in Cross-sections Below and Above the Mixer

Cross-section *I* (below the mixer)

	Downward				Upward	
d/D —	n min^{-1}	$\dot{E}_{11}$[a] Nm s^{-1}	$\dot{V}_{c1}$[b] . 10^3 $\text{m}^3\,\text{s}^{-1}$	K_{c1} —	$\dot{V}_{c2}$. 10^3 $\text{m}^3\,\text{s}^{-1}$	K_{c2} —
1/5	900	2·435	− 9·880	−3·490	9·380	3·215
	1 100	4·285	−12·445	−3·480	12·040	3·420
	1 300	7·460	−14·585	−3·440	13·465	3·180
1/4	600	1·935	− 9·950	−2·560	9·745	2·545
	900	6·750	−14·875	−2·590	13·280	2·310
	1 100	13·370	−18·315	−2·615	17·385	2·485
1/3	350	1·605	−10·645	−2·030	10·005	1·915
	500	4·370	−15·730	−2·100	14·020	1·900
	575	6·450	−18·185	−2·110	18·490	2·145

Cross-section *II* (above the mixer)

		Downward			Upward		
d/D —	n min^{-1}	$\dot{E}_{I12}$ Nm s^{-1}	$\dot{V}_{c2}$. 10^3 $\text{m}^3\,\text{s}^{-1}$	K_{c2} —	$\dot{E}_{II1}$ Nm s^{-1}	$\dot{V}_{c1}$. 10^3 $\text{m}^3\,\text{s}^{-1}$	K_{c1} —
1/5	900	−0·260	− 9·220	−3·155	−0·080	9·740	3·370
	1 100	−0·645	−11·780	−3·295	−0·165	11·910	3·330
	1 300	−0·880	−14·530	−3·420	−0·225	13·670	3·230
1/4	600	−0·150	− 9·145	−2·385	−0·030	9·885	2·580
	900	−0·755	−13·270	−2·310	−0·210	14·145	2·460
	1 100	−1·110	−18·040	−2·575	−0·355	18·540	2·645
1/3	350	−0·275	− 9·810	−1·900	−0·100	10·335	1·970
	500	−0·890	−15·600	−2·080	−0·200	15·394	2·075
	575	−1·510	−18·025	−2·090	−0·270	17·460	2·025

[a] Accuracy of determination of quantity E_{ij} [$i = I, II$, $j = 1, 2$] from results of experiments is $\pm 0{\cdot}05 N\,\text{m s}^{-1}$. [b] Accuracy of determination of quantity $\dot{V}_{cj}$ [$j = 1, 2$] from results of experiments is $\pm 0{\cdot}25 . 10^{-3}\,\text{m}^3\,\text{s}^{-1}$.

From values of constants a_i and C_i $(i = 1, 2)$ given in Table III can be concluded with regard to their dispersion variance that for cross-sections *I* and *II* they equal. From comparison with results of determination of the total volumetric flow rate in a given direction in the cross-section 40 mm far from the lower end of the mixer blades[6], the following relation was obtained

$$K_c = 0{\cdot}373(d/D)^{-1{\cdot}02} \tag{33}$$

in which at the same conditions as in this work the value of exponent a_i in Eq. (*31*) to (*33*) is practically the same, but the value of coefficients C_i in Eq. (*31*) and (*32*) is significantly higher than is the value of this coefficient in Eq. (*33*). This fact can be explained by a successive reversal of liquid layers streaming from the rotor region downward toward the bottom into the upward stream toward the liquid surface so that the total liquid flow rate in a given cross-section decreases with the increasing axial distance from the mixer toward the bottom. These assumption are in agreement with the results of measurements of the force effects of liquid mixed by an axial mixer[9] on the vessel bottom.

From profiles of axial component of the local velocity vector were drawn the curves representing the boundaries of cross-sections of partial volumetric flow rates between the planes *I* and *II* defined by

$$2\pi \int_0^{r_I} [\overline{w}_{ax}(r)]_{I1}\, r\,\mathrm{d}r = 2\pi \int_0^{r_{II}} [\overline{w}_{ax}(r)]_{II1}\, r\,\mathrm{d}r = \dot{V}_{cp}\,,$$

$$[r_I \in \langle 0;\, r_{I1}\rangle;\; r_{II} \in \langle 0;\, r_{II1}\rangle]\,, \tag{34}$$

for the flow directed downward and for the flow directed upward by

$$2\pi \int_{r_{I1}}^{r_I} [\overline{w}_{ax}(r)]_{I2}\, r\,\mathrm{d}r = 2\pi \int_{r_{II2}}^{r_{II}} [\overline{w}_{ax}(r)]_{II2}\, r\,\mathrm{d}r = \dot{V}_{cp}\,,$$

$$[r_I \in \langle r_{I1};\, D/2\rangle;\; r_{II} \in \langle r_{II1};\, D/2\rangle]\,. \tag{35}$$

In Fig. 7 are drawn the curves defined by Eq. (*34*) and (*35*) with the partial flow rates $\dot{V}_{cp}$ given in a dimensionless form (related to the product nd^3) denoted K_{cp} [see Eq. (*22*)]. The drawn segments represent the streamlines through the considered system[10] and thus by their use the field of streamlines can be described. The dashed line is a connecting line of points of flow reversals in cross-sections *I* and *II*; thus it represents a geometrical locus of points of zero velocity *i.e.* the projection of conical surface, over which mass is not transferred and practically, nor the energy. The dash and dot line represents the boundary of region through which is flowing the liquid volume leaving per unit of time blades of a rotating mixer, *i.e.* so-called pumping capacity of the mixer[4,8]. From the figure it follows that the liquid volume flowing through the mixer plane per unit of time in the upward and dowanward direction is greater than the volumetric mixer pumping capacity itself, by the entrained or induced flow[6,8,11], which originates by momentum transfer through turbulent or viscosity friction between the liquid flowing through the region of rotating mixer and charge which is surrounding it. The region of direct mechanical action of the mixer on the charge is therefore not limited by the considered cylinder circumcircled to the rotating mixer, but by a frustum of a cone of volume V_m whose surface is formed by points of zero velocity or of reversal of flow direction from downward to upward. Exactly is the body surface of volume V_m an area of second order whose projection into the vertical plane passing through the axis of the mixed system is a curve and not a straight line in the same way as the streamlines over the volume V_m defined

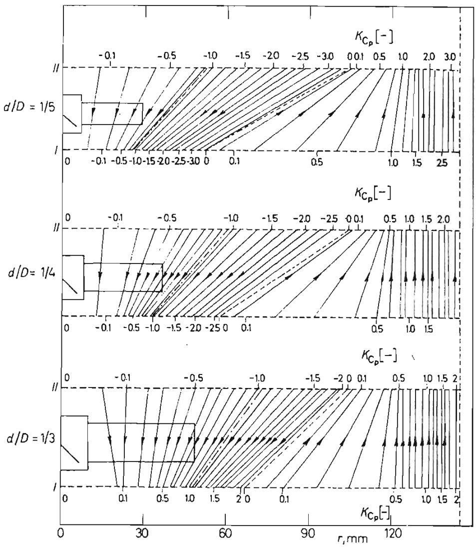

FIG. 7

Streamline Field in Rotor Region of Mixer with Inclined Blades

Values K_{cp} in cross-section *I* for flow downwards ($d/D = 1/3$) are negative.

by Eq. (*34*) and (*35*). The suggested model is a linear approximation of actual conditions and corresponds to means of the used experimental technique.

Flow of Energy in the Mixer Plane

From the determined profiles of the total pressure (measured* by the Pitot tube) representing energy E_{ij} of the unit volume in the liquid flowing through the i-th cross-section and from the axial component of the local velocity vector, the flow rates of energy upward and downward in cross-sections *I* and *II* defined by relations (*2*), (*3*), (*5*), and (*6*) were calculated by numerical integration. Results of the mentioned calculations are given in Table II. The referential plane for geometric height of the i-th cross-section is a horizontal plane passing through the mixer centre (symmetry plane of the mixer). Value of the energy flow rate in cross-section *I* directed upward and in cross-section *II* in both directions, is negative. Due to suction effect of the mixer, there appears in respect to the cited referential plane in the mentioned cross-sections a negative pressure, and the energy values E_{ij} of volume unit in the liquid flow are negative. The absolute value of the energy flow rate through cross-section *I* directed downward is by one order greater than the energy flow rates in remaining parts of the studied cross-sections. This shows, among others, that the flow streaming from the blades of a rotating mixer has the highest energetic level: it comprises mechanical energy which is dissipated in the charge into heat.

Hydraulic Characteristics of the System with Axial Mixer and Radial Baffles

From the determined volumetric and energy flow rates at the inlet and outlet from the rotor region, was calculated by relation (*12*) the energy e_{sp} supplied by the mixer to the unit of flowing liquid volume and, further, the power-output N_t, total height h, and the hydraulic efficiency η_h of the mixer by relations (*14*), (*15*), and (*16*). From so obtained characteristics were calculated the other dimensionless quantities: the dimensionless power-output of the mixer Eu_t, and the dimensionless total mixer height H by relations (*17*) and (*18*). All thus calculated quantities are given in Table IV. Since the studies were made at different values of relative mixer and vessel size d/D the effect of this quantity on the calculated dimensionless characteristics of the mixer was also studied. By the least square method were calculated parameters of power relations

$$\mathrm{Eu}_t = C_3(d/D)^{a_3}\,, \tag{36}$$

$$\mathrm{H} = C_4(d/D)^{a_4} \tag{37}$$

* Total pressure at a given point was always taken by that tube of the probe whose axis formed with the local velocity vector an angle less than 15° [6].

and

$$\eta_h = C_5(d/D)^{a_5}\,. \tag{38}$$

These parameters are, together with estimates of their standard deviation, given in Table III. It can be concluded on their basis that the dimensionless total height of the mixer depends very much on relative size of the mixer while the hydraulic efficiency and dimensionless power-output of the mixer depend only on the fifth

TABLE III

Effect of Relative Mixer and Vessel Size on Flow Rate and Hydraulic Characteristics of the Mixed System

Quantity	a_i	σ_{a_i}	C_i	σ_{C_i}
K_{cI}	$-0·959$	0·052	0·696	0·031
K_{cII}	$-0·954$	0·046	0·686	0·026
Eu_t	$-0·188$	0·125	0·885	0·188
H	0·745	0·046	1·224	0·110
η_h	$-0·198$	0·062	0·511	0·102
$Eu_t/(V - V_m)$	$-0·178$	0·090	48·32	5·66
Eu_{zI}/V_I	$-0·304$	0·028	130·9	15·12
Eu_{zII}/V_{II}	$-1·01$	0·012	28·80	3·11

TABLE IV

Hydraulic Characteristics of the System with an Axial Mixer and Radial Baffles

d/D —	n min^{-1}	$e_{sp}\,.\,10^{-2}$ $N\,m^{-2}$	N_t $N\,m\,s^{-1}$	$h\,.\,10^2$ m	Eu_t —	$H\,.\,10^1$ —	η_h %
1/5	900	2·825	2·695	2·88	1·22	3·73	71·6
	1 100	4·070	4·925	4·15	1·23	3·57	72·4
	1 300	5·725	8·345	5·83	1·18	3·64	69·4
1/4	600	2·185	2·085	2·23	1·10	4·17	64·6
	900	5·345	7·505	5·45	1·12	4·53	65·8
	1 100	7·765	14·480	8·11	1·18	4·51	63·4
1/3	350	1·840	1·880	1·875	1·12	5·57	65·8
	500	3·360	5·260	3·42	1·08	5·24	63·4
	575	4·395	7·360	4·48	1·07	5·17	62·3

root of the mentioned ratio. The found hydraulic characteristics of the mixed system depend, of course, on the choice of cross-section *I* and *II* below and above the mixer and of credible results will be obtained the closer will be the mentioned cross-sections to the lower or upper edge of the mixer blades. Therefore, these cross-sections were chosen so close to the mixer as allowed the used experimental technique. The hydraulic efficiency of the studied mixer type makes 60 to 70% in dependence on relative size of the mixer. It means that within the space of the rotor region V_m (which takes 3–4% of the charge volume) dissipates 30–40% of the mixer power-input and in the remaining part of the charge 60–70% of the power-input. Similarly as

TABLE V

Spacial Distribution of Energy Dissipation in the System with Axial Mixer and Radial Baffles $V_I = 5{\cdot}32 . 10^{-3}$ m^3; $V_{II} = 13{\cdot}20 . 10^{-3}$ m^3; $Eu_m = 1{\cdot}70^a$, $Eu_m/V = 87{\cdot}8$ m^{-3}; $V_m = 0{\cdot}64 . 10^{-3}$ m^3.

d/D —	n min^{-1}	$\varepsilon_I . 10^{-2}$ N m^{-2} s^{-1}	$\varepsilon_{II} . 10^{-1}$ N m^{-2} s^{-1}	Eu_{zI} —	$Eu_{zII} . 10^2$ —	Eu_{zI}/Eu_t %	Eu_{zII}/Eu_t %
1/5	900	4·735	1·450	1·13	8·70	92·7	7·3
	1 100	8·365	2·365	1·17	6·50	94·1	5·9
	1 300	14·470	4·365	1·09	8·80	92·3	7·7
1/4	600	3·700	1·210	1·00	10·20	90·9	9·1
	900	13·100	4·130	1·03	8·80	92·0	8·0
	1 100	25·680	8·135	1·09	9·20	92·4	7·8
1/3	350	3·205	1·325	1·01	11·10	90·0	10·0
	500	8·585	5·250	0·94	13·90	87·4	12·6
	575	12·720	7·075	0·94	13·00	88·2	11·8

d/D —	n min^{-1}	$Eu_t/(V - V_m)$ m^{-3}	$(Eu_m - Eu_t)/V_m$ m^{-3}	Eu_{zI}/V_I m^{-3}	Eu_{zII}/V_{II} m^{-3}
1/5	900	65·5	820	212	6·60
	1 100	66·4	805	220	5·92
	1 300	63·6	890	205	6·47
1/4	600	59·3	970	188	6·42
	900	60·5	935	194	6·37
	1 100	63·6	840	205	6·57
1/3	350	60·6	840	190	8·11
	500	58·6	900	177	10·53
	575	58·0	910	177	9·86

[a] Taken from paper[12].

with the pumps, the energy losses, caused by eddy motion in the rotor region, are the highest. However, in the case of the mixer it is not always necessary to recommend arrangement of the system so that the value of quantity η_h would be greatest because the quantity $(100 - \eta_h)$ is the measure of mixing effect in the rotor region which in case of gas dispersion or emulsification of immiscible liquids is required to be the largest. For axial mixer types used mostly because of their high pumping efficiency, expressed by ratio $\dot{V}_c/N$, the requirement of the as high as possible hydraulic efficiency can be considered as justified because the greater part of the power-input will be used for formation of the averaged component of the convective flow instead of the fluctuation component which increases the mixing effects in the rotor region itself.

Spacial Distribution of Mechanical Energy Dissipation in the System with Axial Mixer and Radial Baffles

From the determined energy flow rates at the inlet and outlet of volumes V_I and V_{II}, and from the size of these volumes, was calculated by relations (*1*) and (*4*) the energy ε_i $(i = 1{\cdot}2)$ dissipated in the unit of volumes V_I and V_{II} of the charge. From these characteristics was then for the given conditions in the mixed system calculated the dimensionless quantity Eu_{zi} according to Eq. (*21*). The calculated values are for volumes V_I and V_{II} given in Table V. In this Table are also given quantities which characterize spacial distribution of the dissipation rate of mechanical energy in the mixed system. This is first of all the quantity $\mathrm{Eu}_{zi}/\mathrm{Eu}_t$ $(i = I, II)$ characterizing distribution of dissipation of the mixer power-output in the charge volume and, further, quantities characterizing the rate of energy dissipation in the volume unit of various parts of the charge:

1. $\mathrm{Eu}_t/(V - V_m)$, *2.* $(\mathrm{Eu}_M - \mathrm{Eu}_t)/V_m$, *3.* Eu_{zI}/V_I, *4.* Eu_{zII}/V_{II}.

First of these quantities is characterizing the mean dissipation of the mixer power-output in the charge, the next quantities consecutively rates of energy dissipation in the unit of volume V_m, V_I, and V_{II}. In Table III are then given the parameters of regression dependences

$$\mathrm{Eu}_t/(V - V_m) = C_6(d/D)^{a_6}\,, \tag{39}$$

$$\mathrm{Eu}_{zI}/V_I = C_7(d/D)^{a_7}\,, \tag{40}$$

and

$$\mathrm{Eu}_{zII}/V_{II} = C_8(d/D)^{a_8}\,, \tag{41}$$

calculated by the least square method from the data given in Table V. Other dimensionless quantities given in this Table, were not correlated with the ratio d/D because the effect of change of these variables with the change of relative size of the mixer, was not significant due to dispersion of constants of calculated regression dependences. From the calculated results given in the mentioned Tables results that the mixed system is from the view of spacial distribution of mixer power-output dissipation considerably heterogeneous. In the space of the charge below the cross-section *II* reduced by the volume of region V_m, though it occupies approximately 1/4 of the total volume, dissipates practically 90% of the energy supplied to the charge by the mixer and in the space V_{II} above the cross-section *II* dissipates only 10% of the mixer power output though this space forms almost 2/3 of the charge volume. The differences between the considered volumes are more obvious when we compare the rate of energy dissipation in the unit of each of these volumes. This quantity is in the volume V_I approx. 20–35 times greater than in the volume above the mixer (in dependence on relative size of the mixer) and in the rotor region is the rate of energy dissipation in the volume unit more than 4 times greater than in the space V_I.

The explanation of considerable difference between the values of quantities ε_I and ε_{II} can be found in the validity of general hydrodynamic laws on turbulent flow for the considered mixed system as for inst.[13]. Between the rate of energy dissipation in unit of the *i*-th volume and the velocity field at the inlet into the considered volume, holds the relation

$$\varepsilon_i V_i/g = \zeta_i(\overline{w}^2)_{avi} \dot{V}_{c_i}\varrho/2g\,, \tag{42}$$

where the mean kinetic energy of volume unit of the stream, entering through the *i*-th cross-section into the *i*-th volume, is defined for the volume V_I and cross-section *I* by relation

$$(\overline{w}^2)_{avI}/2g \equiv 2\pi \int_0^{r_{I1}} [\overline{w}_{ax}(r)]_{I1}^2\, r\, dr/2\pi g r_{I1}^2 \tag{43}$$

and for volume V_{II} and cross-section *II* by relation

$$(\overline{w}^2)_{avII}/2g \equiv 2\pi \int_{r_{II1}}^{D/2} [\overline{w}_{ax}(r)]_{II2}^2\, r\, dr/2\pi g(D^2/4 - r_{II1}^2)\,. \tag{44}$$

If we assume that for the friction factor ζ_i (i = I, II) (for change of flow direction by 180°) in the volumes V_I and V_{II} holds

$$\zeta_I \approx \zeta_{II}\,, \tag{45}$$

then the ratio of rates of energy dissipation in unit of volumes V_I and V_{II} equals — with regard to relation (*10*) to

$$\varepsilon_I/\varepsilon_{II} = V_{II}(\overline{w}^2)_{avI}/V_I(\overline{w}^2)_{avII}\,. \tag{46}$$

The ratio of quantities ε_I and ε_{II} was calculated from experimentally determined velocity profiles $[\overline{w}_{ax}(r)]_{I1}$ and $[\overline{w}_{ax}(r)]_{II2}$, and there was obtained a good agreement with the results based on the

TABLE VI

Comparison of Experimental and Theoretical Ratios of Dissipation Rates of Mechanical Energy in Volumes V_I and V_{II}

d/D	$(\varepsilon_I/\varepsilon_{II})_{exp}$	$(\varepsilon_I/_{II})_{th}$
1/5	34·40	38·00
1/4	31·30	28·30
1/3	19·50	17·20

data given in Table V (see Table VI, where are also, for given ratios d/D, written arithmetic means of experimental values, calculated from the values written in Table V). Spacial distribution of rate of mechanical energy dissipation in volume unit of the mixed charge is thus in a very close relation with the flow pattern of the system and conclusions can be made concerning these quantities if we know the field of velocity vector in the system.

At this point it is necessary to explain why the study of spacial distribution of energy dissipation was made here at such extreme relative distance of the mixer ($h/D = 1/4$) above the bottom. This distance was chosen so as to determine from the experimental results of the velocity field quantitatively the relation between the axial force and the velocity field in vicinity of the bottom[9]. Measurements of axial force acting on the bottom could not be made then with sufficient accuracy for all the used sizes of the mixer at a distance greater than $h/D = 1/4$ and measurement of velocity fields in the mixed charge by Pitot tube was very time-consuming to enable to change the mixer distance from the bottom.

From the obtained results can be estimated that the rate of mass and heat transfer in the mixed charge will be larger in the space of the rotor region and below it while the space above the mixer plane can be considered as practically still region. This fact has already been directly proved experimentally by a study of rates of mass[14] and heat[15] transfer in the charge mixed by an axial mixer in a vessel with radial baffles in the turbulent flow regime. Rate of the mentioned processes was always found significantly greater in the space V_I than in V_{II}. The effect of change of the relative mixer size on the heterogeneity change of the rate of energy dissipation in the mixed system is related with the magnitude of radius of the lower base of rotor region V_m, while radius of the upper base of volume V_m was found independent of the magnitude of ratio d/D. Differences in the spacial distribution of mechanical energy dissipation in the mixed charge are therefore reduced by the use of relatively greater mixer and by situating the mixer higher above the vessel bottom at constant ratio H/D. As is obvious from Eq. (*46*) for such processes, where is required spacial homogeneity of the rate of mass and heat transfer in the charge can be considered as suitable a relatively greater mixer situated farther from the vessel bottom, while for processes where is required greater transfer rate in the space at the bottom instead in the space above the mixer plane a relatively smaller mixer, situated closer to the vessel bottom is more suitable.

The authors thank for valuable advices and comments to Dr V. Kudrna, Department of Chemical Engineering, Institute of Chemical Technology, Prague

LIST OF SYMBOLS

a_i exponents
b radial width of baffle (m)
b exponent in Eq. (*27*)
C_i multiplication constants
D mixer diameter (m)
d vessel diameter (m)
E_{ij} energy of volume unit in liquid stream through i-th cross-section vertically downwards ($j = 1$) or vertically upwards ($j = 2$) (N m^{-2})
$\dot{E}_{ij}$ flow rate of energy ($j = 1$) into the i-th insulated volume or from the considered volume ($j = 2$) (N m s^{-1})
e_{sp} energy supplied by the mixer to unit volume of liquid in volumetric flow rate $\dot{V}_c$ in region V_m (N m^{-2})
h total height of mixer (m)
h_2 height of mixer above the vessel bottom (m)
j summation index
K_i constant in regression Eq. (*27*) and (*28*)
k_i slope of velocity profile in I and II ray (m^{-1})
N power-input of the mixer (N m s^{-1})
N_t power-output of the mixer (N m s^{-1})
n rotational speed of the mixer (s^{-1})
p_{st} local mean value of static pressure (N m^{-2})
q_i constant in regression Eq. (*26*) and (*29*)
r radial coordinate (distance from system axis) (m)
r_{i1} radial coordinate in i-th cross-section, where the flow direction is reversed (m)
r_{ic} radial coordinate in i-th cross-section, where the profile of axial velocity component changes its course (m)
T time interval (s)
V_i volume of i-th system ($i = I, II$, m) (m^3)
$\dot{V}_c$ total volumetric flow rate of the mixed liquid in mixer plane directed vertically upward or downward (m^3 s^{-1})
$\dot{V}_{ci}$ total volumetric flow rate of the mixed liquid through the i-th cross-section (m^3 s^{-1})
$\dot{V}_{cp}$ partial volumetric flow rate between cross-sections I and II downward or upward (m^3 s^{-1})
$\bar{w}$ absolute value of local velocity vector, averaged in time (m s^{-1})
$\bar{w}_{ax}$ axial component of quantity $\bar{w}$ (m s^{-1})
$[\bar{w}_{ax}(r)]_{ij}$ radial profile of quantity $\bar{w}_{ax}$ in the i-th cross-section in downward ($j = 1$) or upward ($j = 2$) direction (m s^{-1})
w' absolute value of fluctuation velocity vector (m s^{-1})
$(\bar{w}^2)_{avi}/2g$ mean kinetic energy of volume unit of stream entering through the i-th cross-section into the i-th volume (m)
z_i height of i-th cross-section above the referential horizontal plane (m)
γ angle of blade inclination (deg)
ε_i energy dissipated per unit of time in unit of the i-th volume (N m^{-2} s^{-1})
η dynamic viscosity of mixed charge (kg m^{-1} s^{-1})
ϱ density of mixed charge (kg m^{-3})

τ time (s)
σ_a estimation of standard deviation of quantity a
φ angle between the local velocity vector and horizontal plane (deg)
ψ angle between the local velocity vector and vertical plane (deg)
$\mathrm{Eu}_M \equiv N/\varrho n^3 d^5$ Euler (power-input) number
$\mathrm{Eu}_t \equiv N_t/\varrho n^3 d^5$ dimensionless mixer power-output
$\mathrm{Eu}_{zi} \equiv \varepsilon_i V_i/n^3 d^5$ dimensionless quantity of energy dissipated per unit of time in i-th volume of mixed system
$\mathrm{H} \equiv gh/n^2 d^2$ dimensionless total height of mixer
$K_c \equiv \dot{V}_c/nd^3$ flow criterion
K_{ci} flow criterion for i-th cross-section
K_{cp} flow criterion for partial flow rate $\dot{V}_{cp}$
$\eta_h = \mathrm{Eu}_t/\mathrm{Eu}_M$ hydraulic efficiency of mixer
ζ_i friction factor at a turn of flow direction for 180° in i-th volume of charge

REFERENCES

1. Bird R. B.: Chem. Eng. Sci. *6*, 123 (1957).
2. Nagata Sh., Yamamoto K., Ujihara G.: Mem. Fac. Eng., Kyoto Univ. *30*, 336 (1958).
3. Nagata Sh., Yamamoto K., Hashimoto K., Naruse Y.: Mem. Fac. Eng., Kyoto Univ. *31*, 260 (1959).
4. Fořt I., Sedláková V.: This Journal *33*, 836 (1968).
5. Fořt I., Kudrna V., Valešová H.: This Journal *36*, 164 (1971).
6. Fořt I., Podivínská J., Baloun R.: This Journal *34*, 959 (1969).
7. Cooper R. G., Wolf D.: Can. J. Chem. Eng. *46*, 94 (1968).
8. Fořt I.: This Journal *32*, 3663 (1967).
9. Fořt I., Eslamy M., Košina M.: This Journal *34*, 3673 (1969).
10. Kito Sh.: J. Fac. Eng., Tokyo Imperial Univ. *20*, No 12, 283 (1936).
11. Porcelli J. V., Garr M. R.: Ind. Eng. Chem. Fundamental *1*, 172 (1962).
12. Kvasnička J.: *Thesis*. Chepos, Brno 1967.
13. Cutter J. A.: A.I.CH.E. J. *12*, 35 (1966).
14. Landau J., Procházka J., Václavek V., Fořt I.: This Journal *28*, 279 (1963).
15. Fořt I., Vlček J., Cink M., Hrubý M.: This Journal *36*, 1546 (1971).
16. Kudrna V., Fořt I., Eslamy M., Cvilink J., Drbohlav J.: This Journal, in press.

Translated by M. Rylek.